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PARAMETRIC STUDY OF THE EFFECTS OF ALTITUDE AND INCLINATION ON THE REGRESSION OF THE ORBITAL PLANE FOR EARTH ORBIT LAUNCH WINDOWS

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PROJECT APOLLO

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FIGURES

PARAMETRIC STUDY OF THE EFFECTS OF ALTITUDE AND INCLINATION ON THE REGRESSION OF THE ORBITAL PLANE FOR EARTH ORBIT LAUNCH WINDOWS

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SUMMARY AND INTRODUCTION

This report presents a parametric study of the effects of the rotation of the orbital plane about the earth's polar axis for a period of 200 days. The study facilitates the planning of launch days and lift-off times for missions in which subsequent vehicles are launched into the plane of a target that has existed for many days. Included in this report are plots which show:

(a) Effects of the nodal regression of low altitude circular orbits having inclinations from 0° to 90°.

(b) Orbital regression rates for circular orbits of altitudes from 100 to 300 n. mi. at selected inclinations from 28.5° to 50°.

(c) Recurrence of the northerly in-plane launch node at selected target altitudes of 100, 200, and 300 n. mi. for inclinations from 28.5° to 50° in terms of time of day for subsequent launch versus days from target lift-off.

(d) Wedge angle curves for various selected inclinations from 28.5° to 50° as a function of time in the window.

(e) Plane change velocity requirement versus wedge angle as a function of circular orbital target altitudes of 100 and 300 n. mi.

ANALYSIS AND RESULTS

The rotation of the orbital plane about the earth's polar axis is due to the oblateness of the earth. This rotation is in a direction opposite to the direction of revolution of the vehicle in its orbit and can be observed in the equatorial plane by the motion of the ascending node (south to north intersection of the orbital and equatorial planes) measured west to east from the Greenwich meridian. In analyzing the effects of the rotation of the orbital plane, the following equation from reference 1 was used:

Rotation of orbital plane (nodal regression) =

$$-\frac{3}{2}J\left[\frac{R_{eq}}{a(1-e^2)}\right]^2 (\cos i)\sqrt{\frac{\mu}{a^3}} \quad \text{(rad/sec)}$$

where J is the coefficient of the potential function, R_{eq} is the equatorial earth radius, μ is the gravitational parameter for the earth, a is the semimajor axis of the orbit, e is the orbital eccentricity, and i is the orbital inclination. The values of J, R_{eq}, and μ used for this study were taken from reference 2 and are as follows:

$$J = 1.08228 \times 10^{-3}$$

$$R_{eq} = 20925738.19 \text{ ft}$$

$$\mu = 1.40765392 \times 10^{16} \text{ ft}^3/\text{sec}^2.$$

The orbital regression rates for various inclinations from 28.5° to 50° as function of circular orbital altitude from 100 to 300 n. mi. can be seen in figure 1. Note that as the altitude increases for a particular inclination, the regression rate decreases.

Figure 2 presents the frequency of the northerly in-plane launch node (first passage of the launch site through target vehicle orbital plane) for various inclinations from 0° to 90° for circular orbital altitudes in the range of 100 to 300 n. mi. As the inclination and altitude increase, the frequency of the northerly in-plane launch node decreases to a minimum, recurring every 23 hr 56 min 4.09 sec (one sidereal day), at 90° inclination where no measurable nodal regression is present.

Figure 3 is presented to emphasize the frequency of the in-plane launch node at the inclinations of 28.5° through 50° which is the range of inclinations presently being considered for future earth orbital missions.

A chronology of the daily repetition of the northerly in-plane launch node is given in figure 4(a) through (f) assuming 4:00 p.m. was the target vehicle lift-off time. Orbital altitudes of 100, 200, and 300 n. mi. and inclinations of 28.5°, 30.0°, 35.0°, 40.0°, 45.0°, and

50.0° were considered. The northerly in-plane launch node was arbitrarily chosen as a reference point in figure 4 to illustrate how this point (the time which is optimum for lift-off of a second vehicle to achieve in-plane conditions with the target orbit) will recur over a period of days to facilitate the launching of subsequent vehicles into the plane of the target orbit. The time of day for recurrence of this node at the launch site is given as a function of days from target lift-off.

It is assumed in figure 4 that the target orbit maintains its original circular orbit of either 100, 200, or 300 n. mi. Consideration must be given to maintaining the orbit, especially in circular orbits less than 200 n. mi. from which reentry into the earth's atmosphere will occur in a relatively short time depending on the effects of drag on the orbit. However, for high altitude circular orbits (200 n. mi. and above) drag has little effect on the time of the recurrence of the node. For example, based on a 1962 standard atmosphere the northerly in-plane node will recur approximately 5 minutes earlier on the 200th day after insertion if a vehicle with ballistic number of 21 (vehicle weight divided by effective area times the drag coefficient) is inserted into a 300-n. mi. orbit.

Also, in figure 4, the lighting conditions at the launch site when the node recurs can be observed. Darkness at the launch site was arbitrarily chosen to be from 6:00 p.m. to 6:00 a.m. In the event multiple subsequent launches are made into the same orbit, these plots can be used to evaluate the time of day of in-plane launches for the various subsequent launches. Target lift-off times other than 4:00 p.m. can be studied simply by assigning the desired time at the pad for the target vehicle lift-off and adjusting the ordinate scale correspondingly. The lighting conditions can be reestablished by observing when 6:00 p.m. and 6:00 a.m. occur on the new scale.

A summary of the time in days that it takes the northerly in-plane launch node to repeat itself at the same time of day as a function of inclination of the circular orbits of 100-, 200-, and 300-n. mi. altitude is presented in figure 5.

Figures 6 and 7 show that the wedge angle and the time between northerly and southerly in-plane launch nodes increase as the inclination increases. Figure 8 shows the plane change velocity requirement with respect to the wedge angle as a function of circular orbital altitudes of 100 and 300 n. mi. Note that at different altitudes, the plane change velocity requirement will vary slightly for the same wedge angle. This is a direct effect of the velocity varying for various altitudes, as given in the following equation:

$$\Delta V = 2V \sin \frac{\delta}{2}$$

where

ΔV = plane change velocity requirement, fps

V = inertial velocity, fps

δ = wedge angle, deg.

CONCLUDING REMARKS

This parametric study provides the user with informative data concerning the orbital regression rate and the daily repetition of the northerly in-plane launch node for various target orbits. In addition, an analysis of the launch window curve in relation to wedge angle at selected inclinations and related curves are presented. Such data can be utilized in the planning of future earth orbital missions.

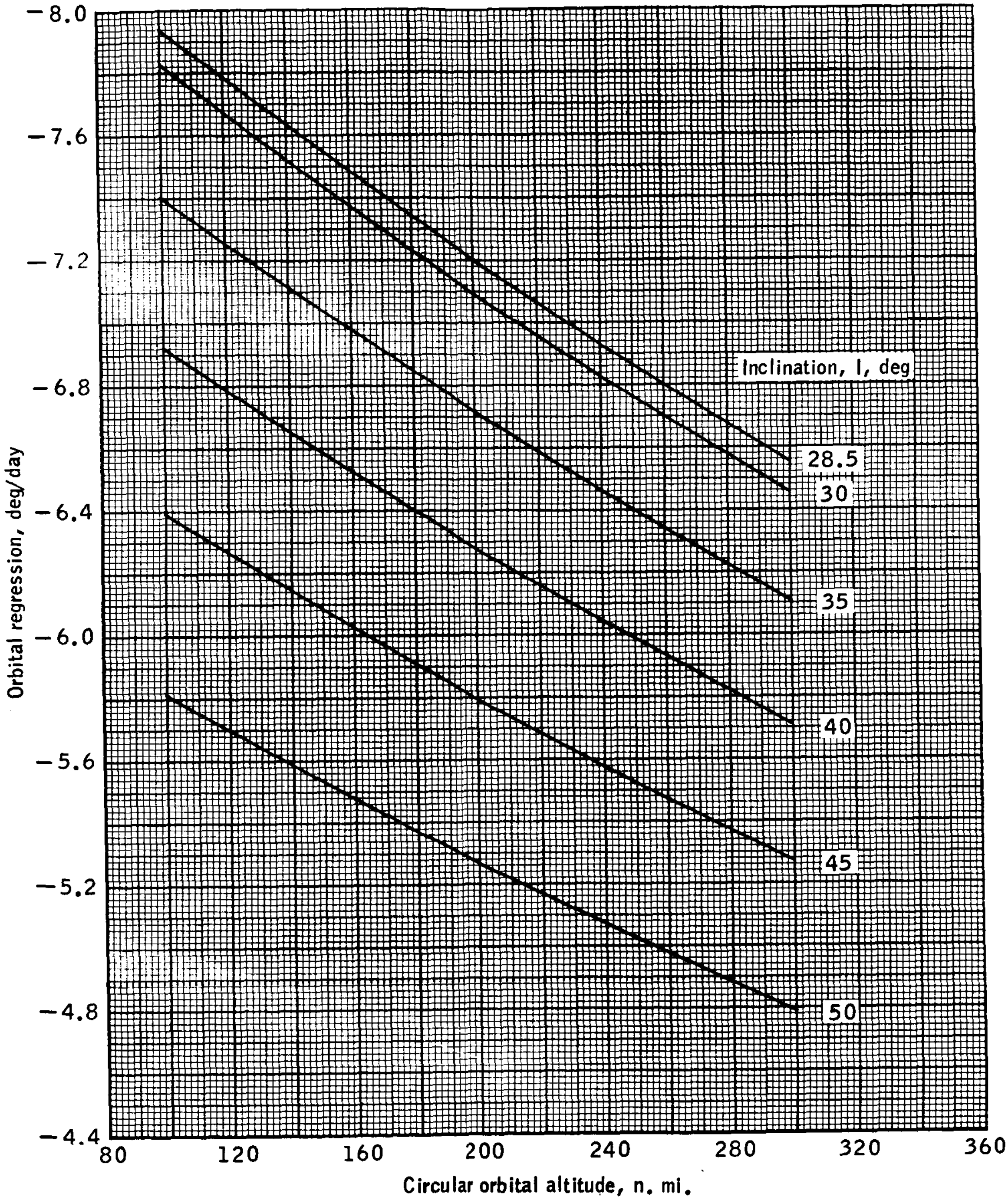


Figure 1.- Orbital regression for various inclinations as a function of altitude.

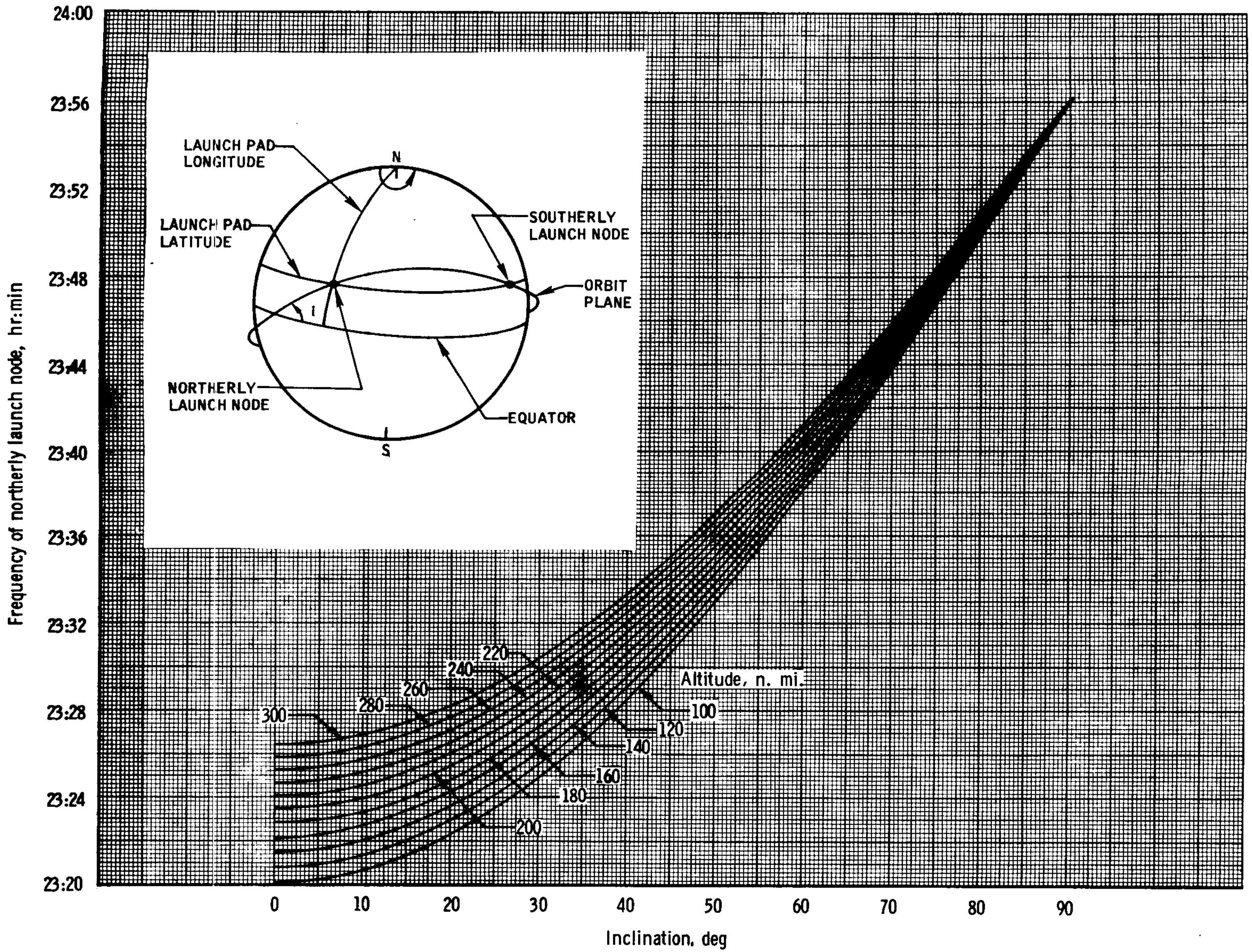


Figure 2.- Frequency of northerly launch node versus inclination (0°-90°) as a function of circular orbital altitude (100-300 n. mi.).

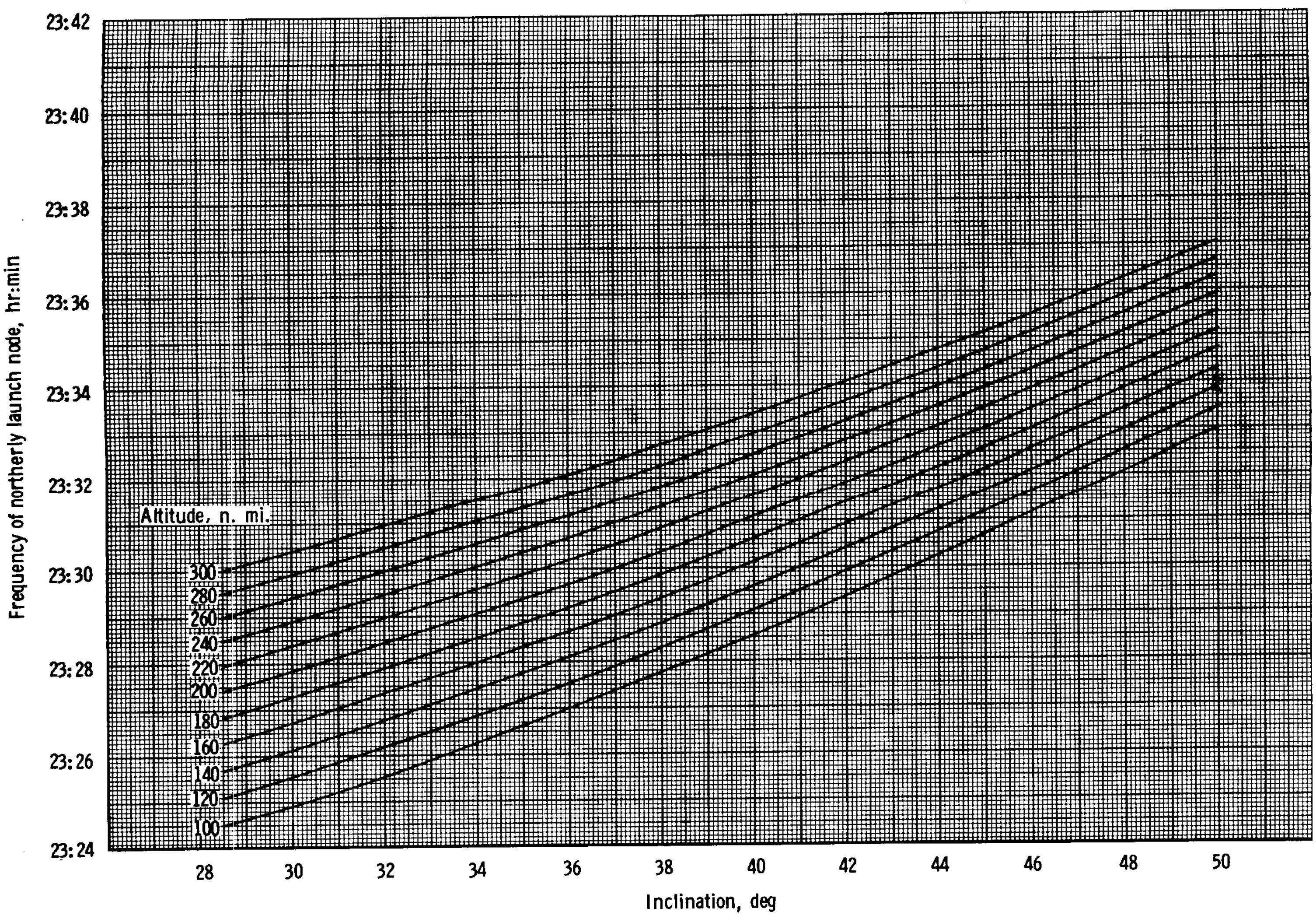
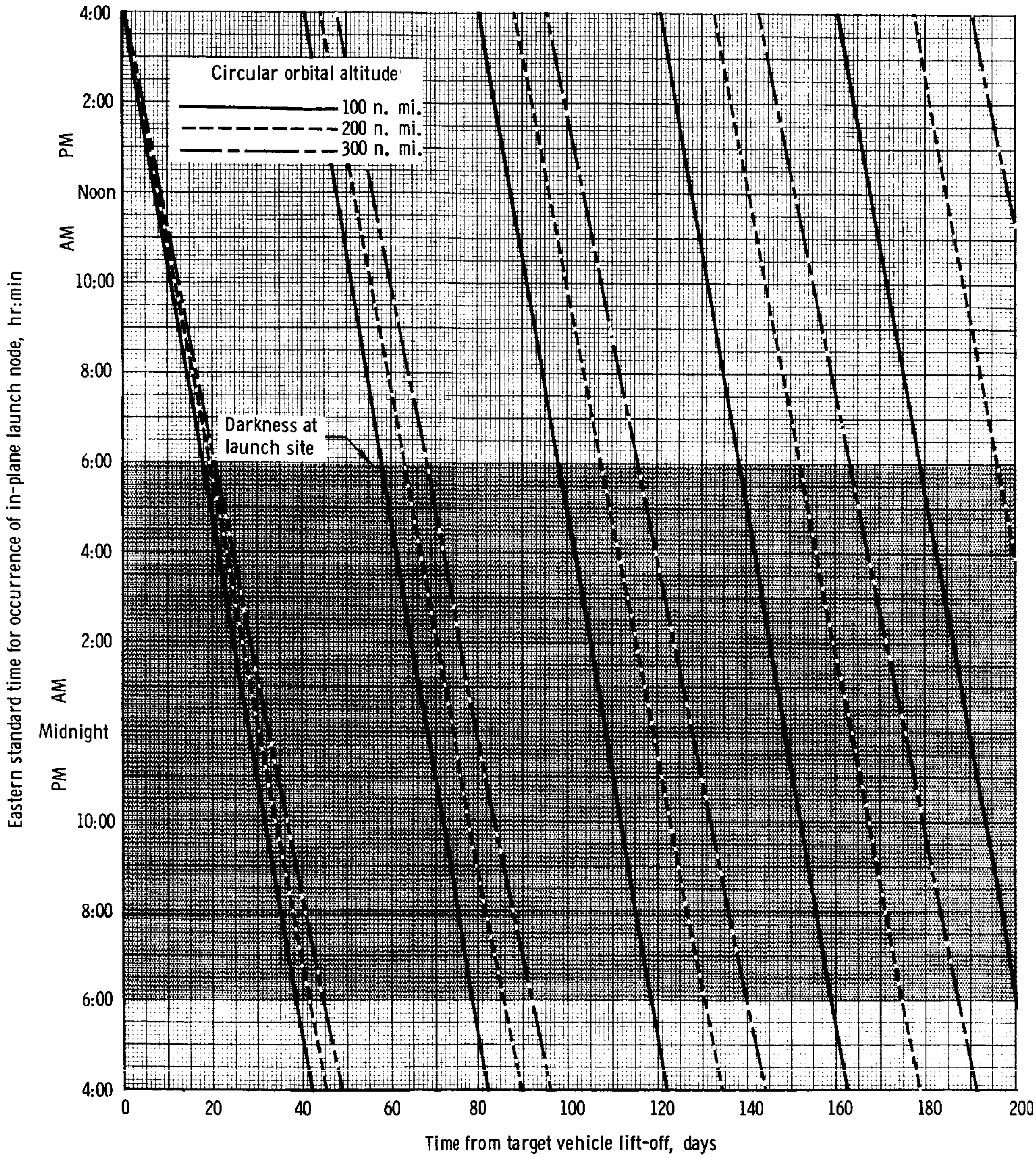
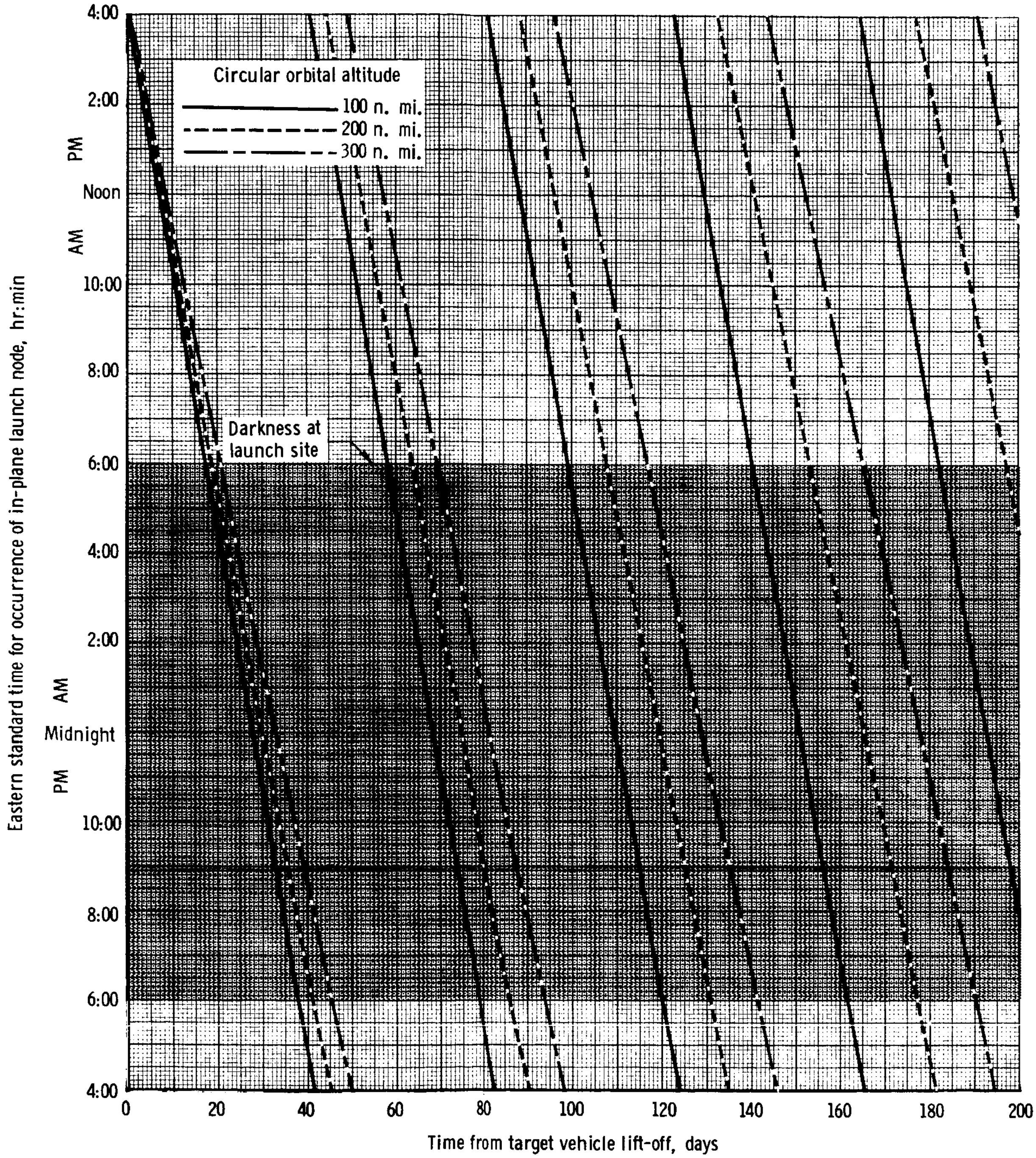


Figure 3. - Frequency of northerly launch node versus inclination (28.5°-50.0°) as a function of circular orbital altitude (100-300 n. mi.)



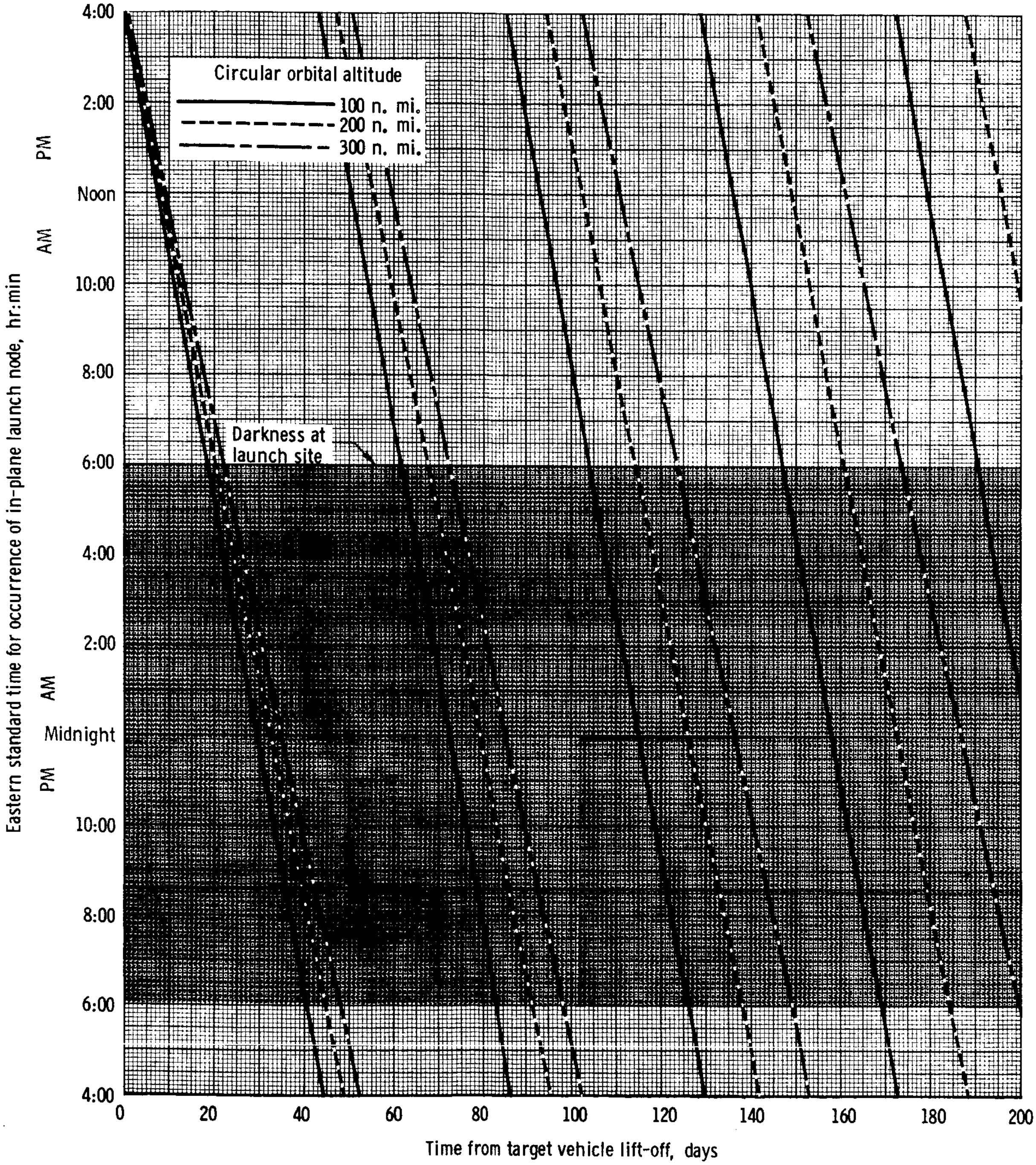
(a) Orbital inclination = 28.5°.

Figure 4. - Time of day of northerly in-plane launch node.



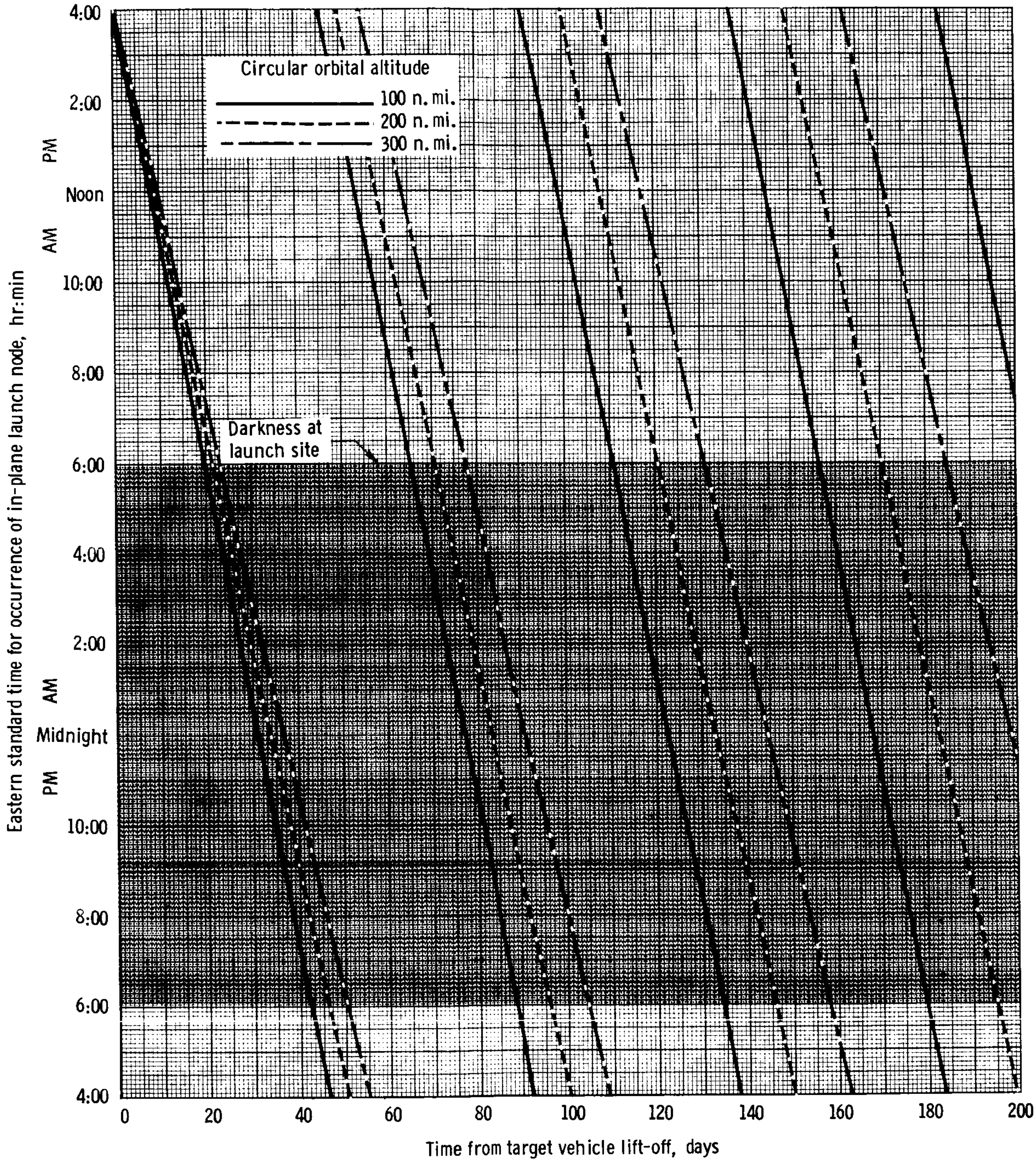
(b) Orbital inclination = 30.0°.

Figure 4.- Continued.



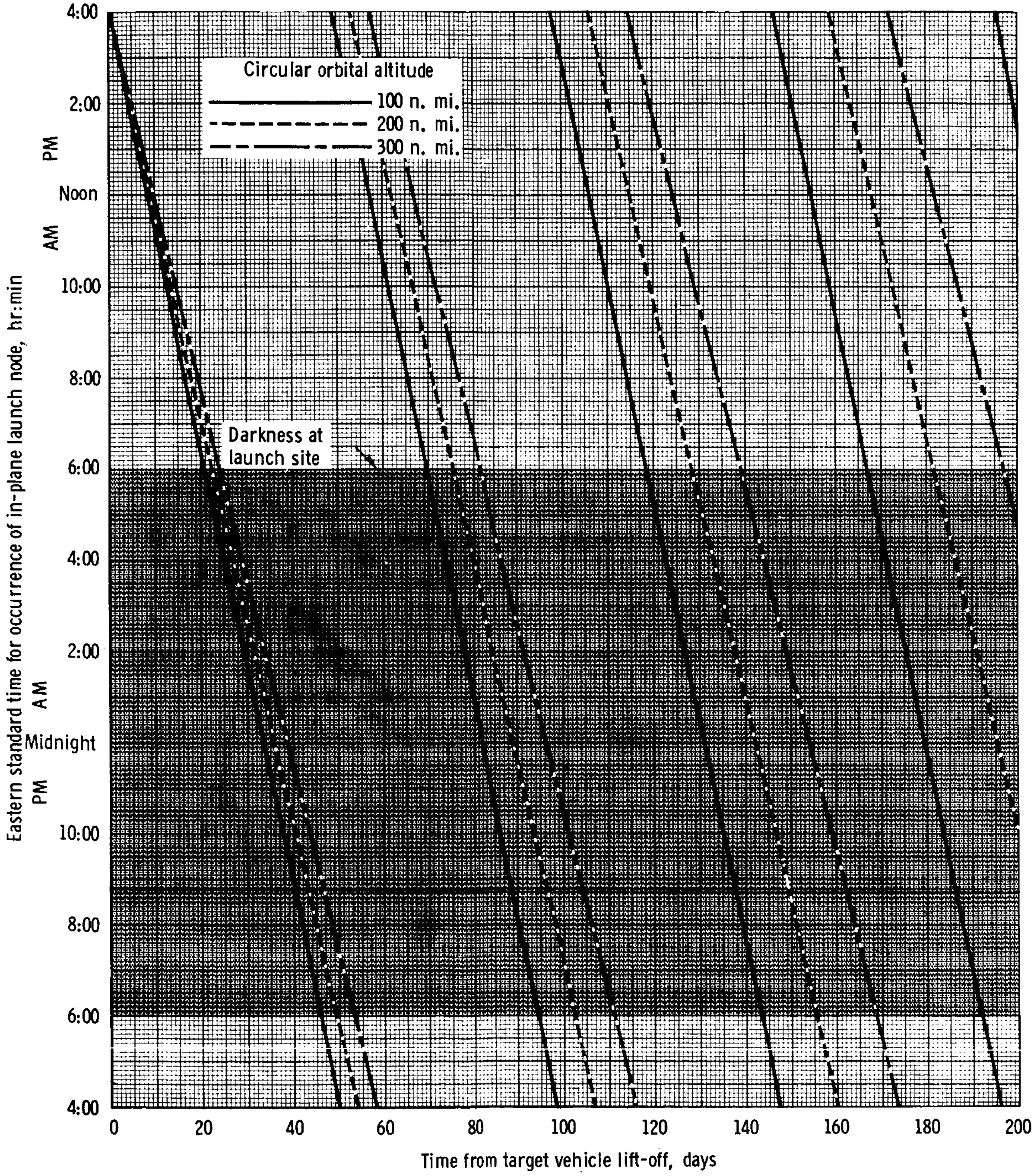
(c) Orbital inclination = 35.0°.

Figure 4.- Continued.



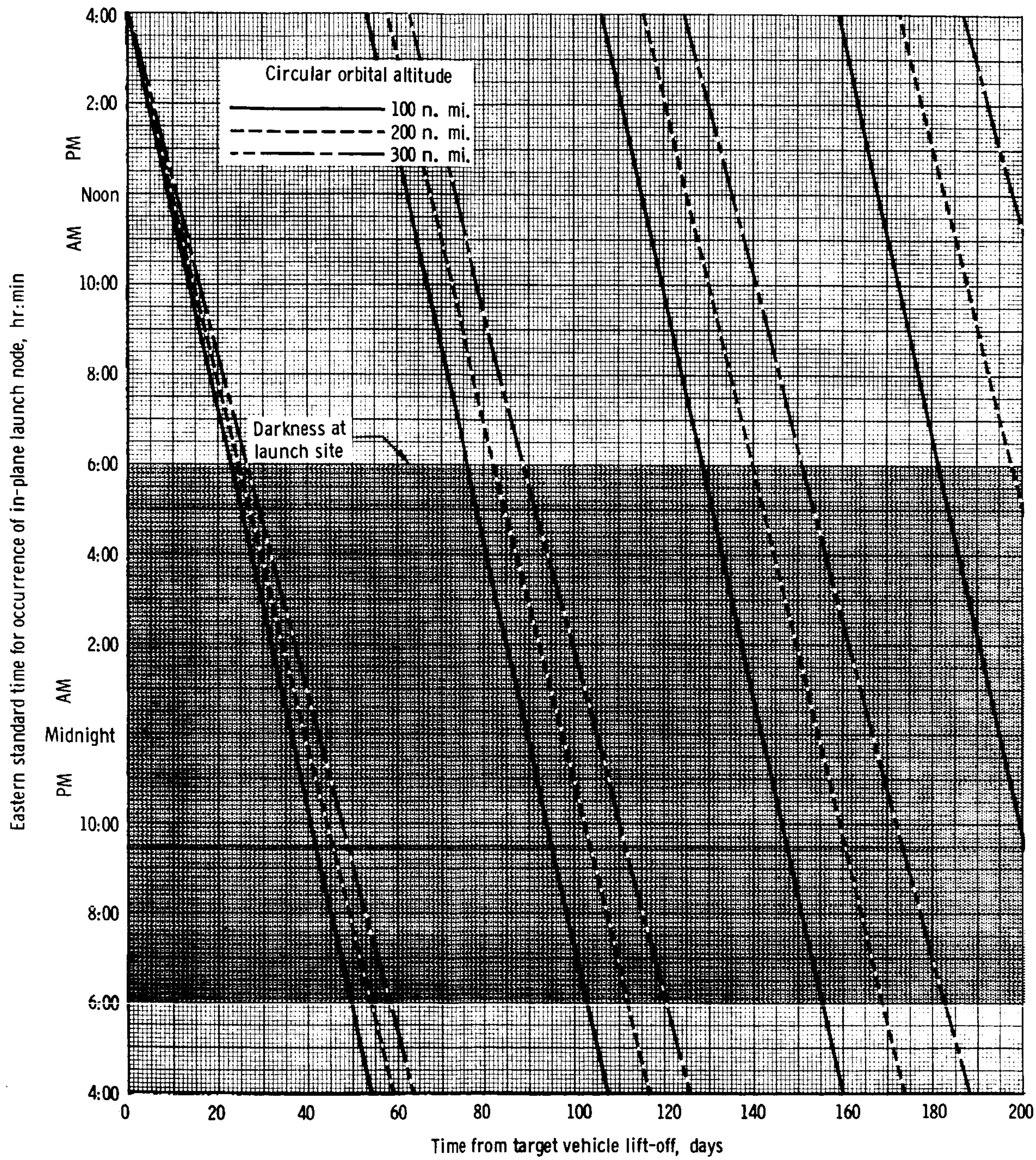
(d) Orbital inclination = 40.0°.

Figure 4. - Continued.



(e) Orbital inclination = 45.0°.

Figure 4. - Continued.



(f) Orbital inclination = 50.0°.

Figure 4.- Concluded.

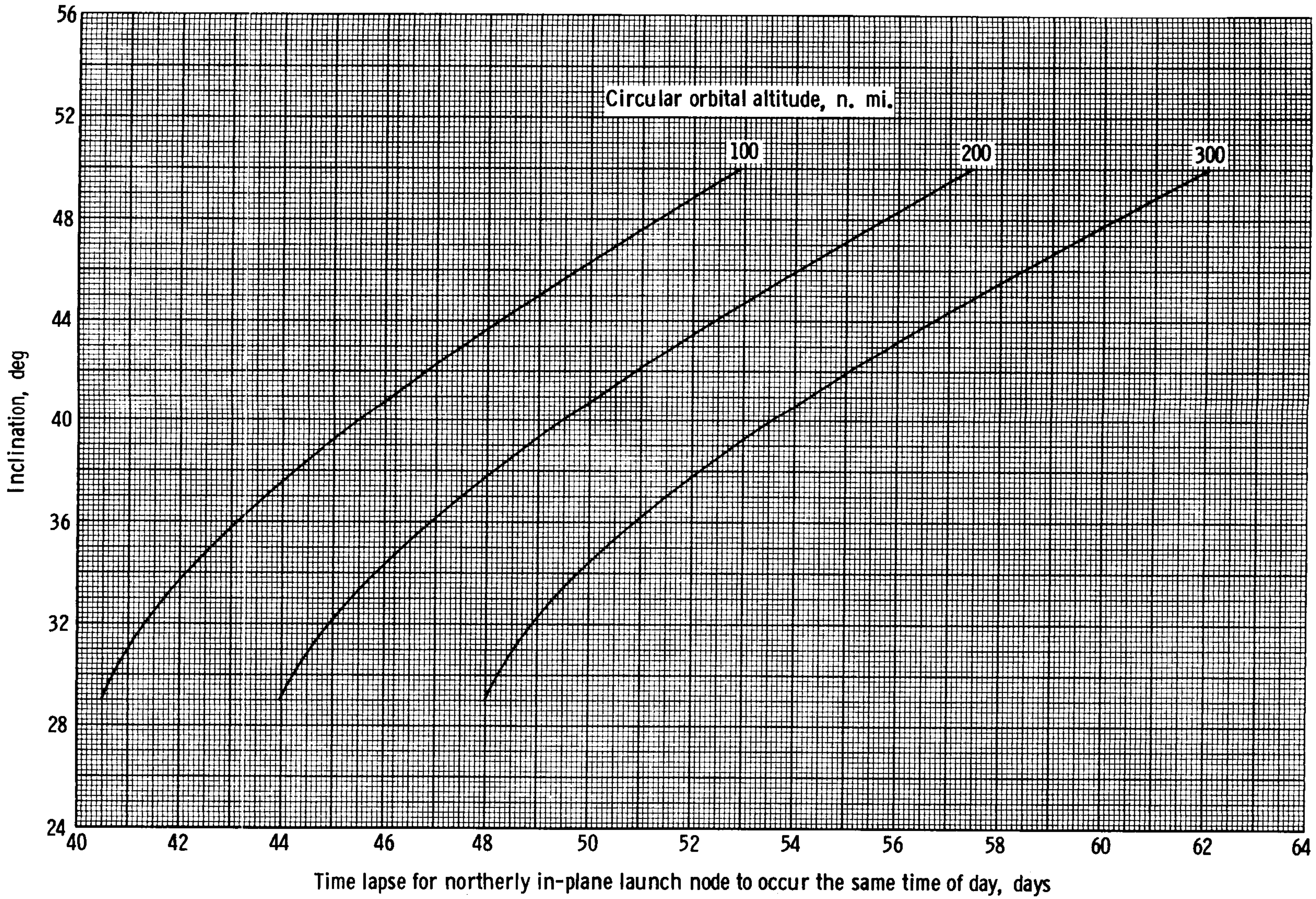


Figure 5.- Effects of inclination as a function of the time required for the northerly in-plane launch node to occur the same time of day for various circular orbital altitudes.

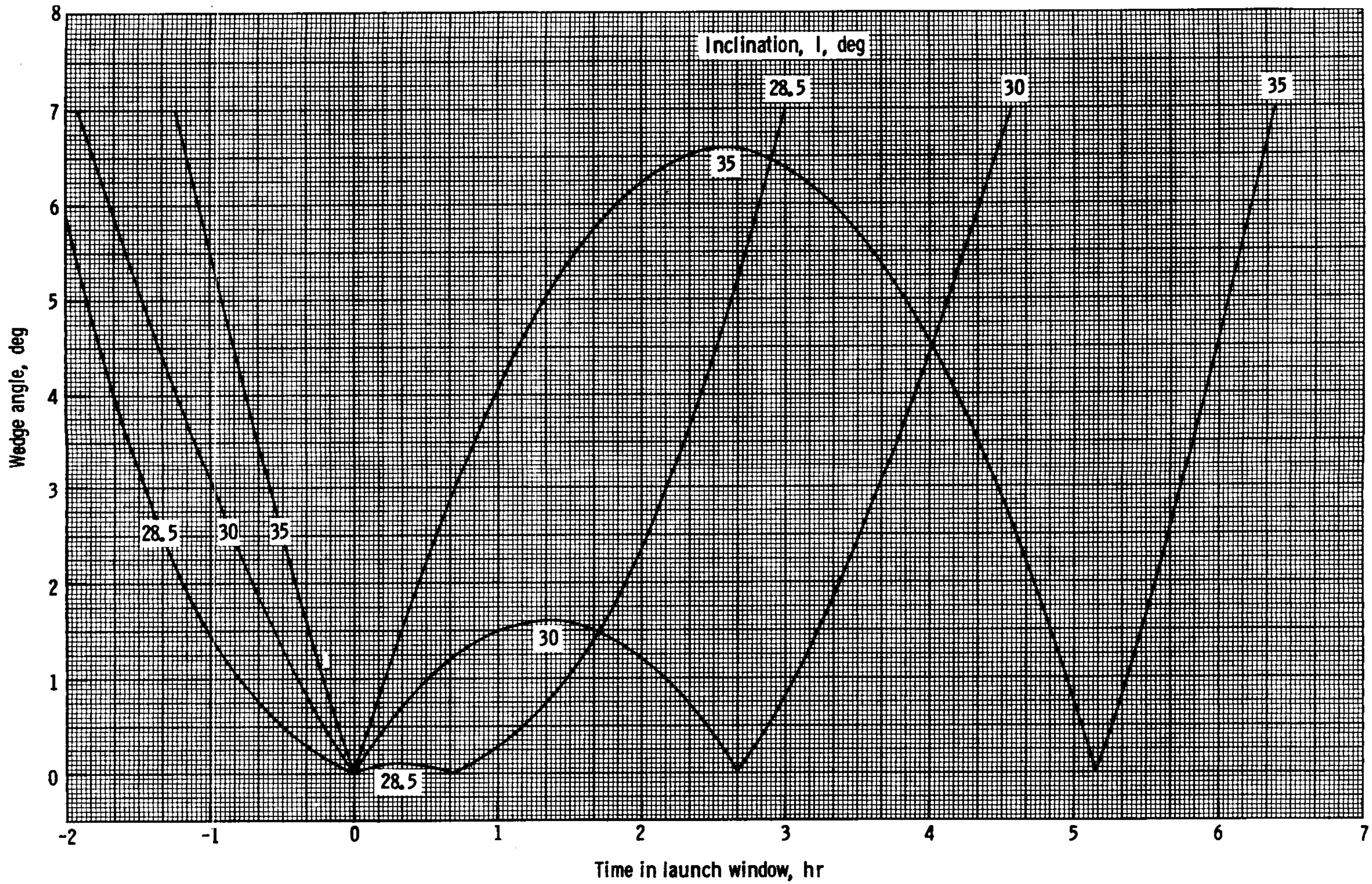


Figure 6.- Wedge angle versus time in launch window for inclinations from 28.5° to 35.0°.

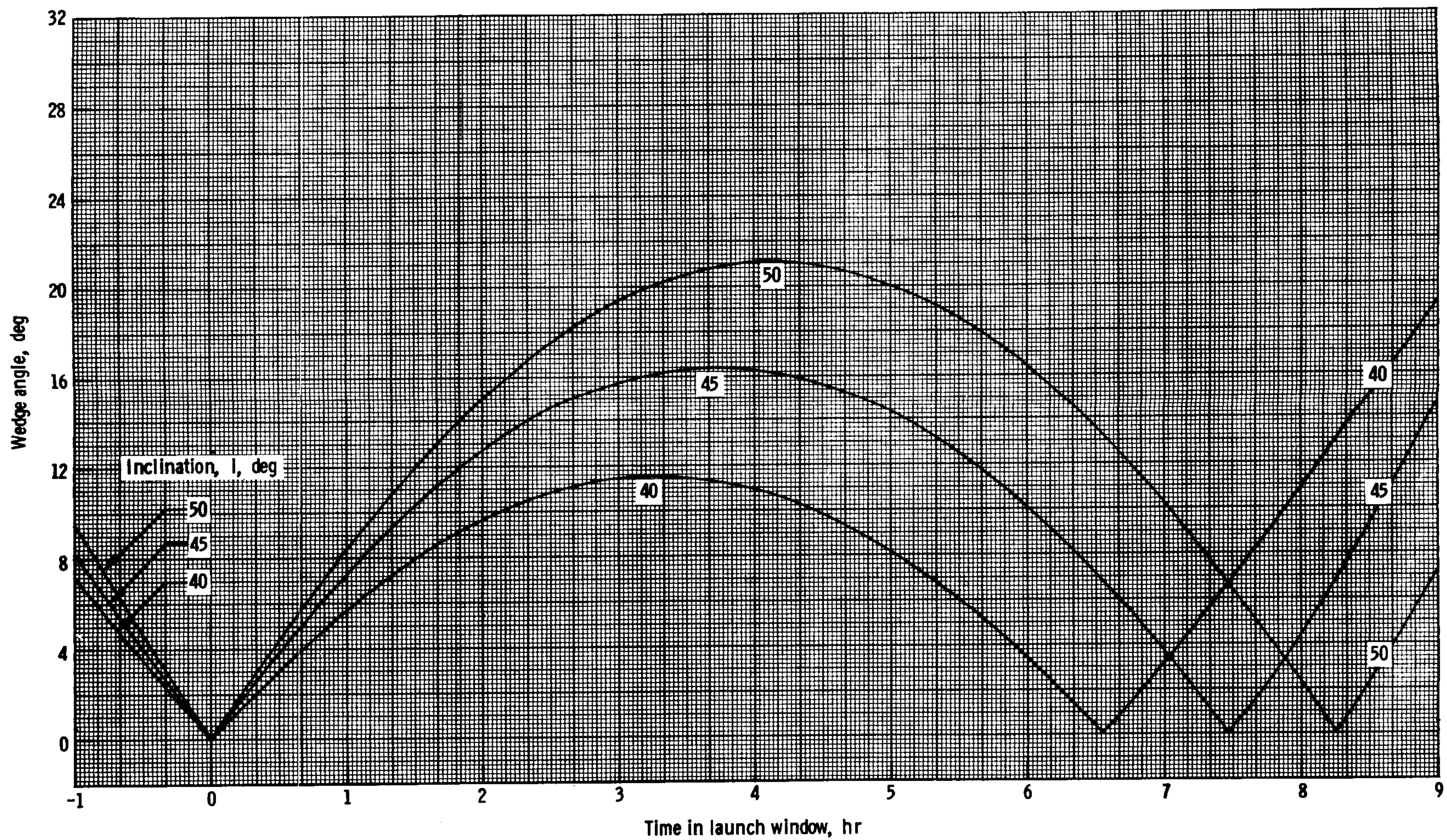


Figure 7.- Wedge angle versus time in launch window for inclinations from 40° to 50°.

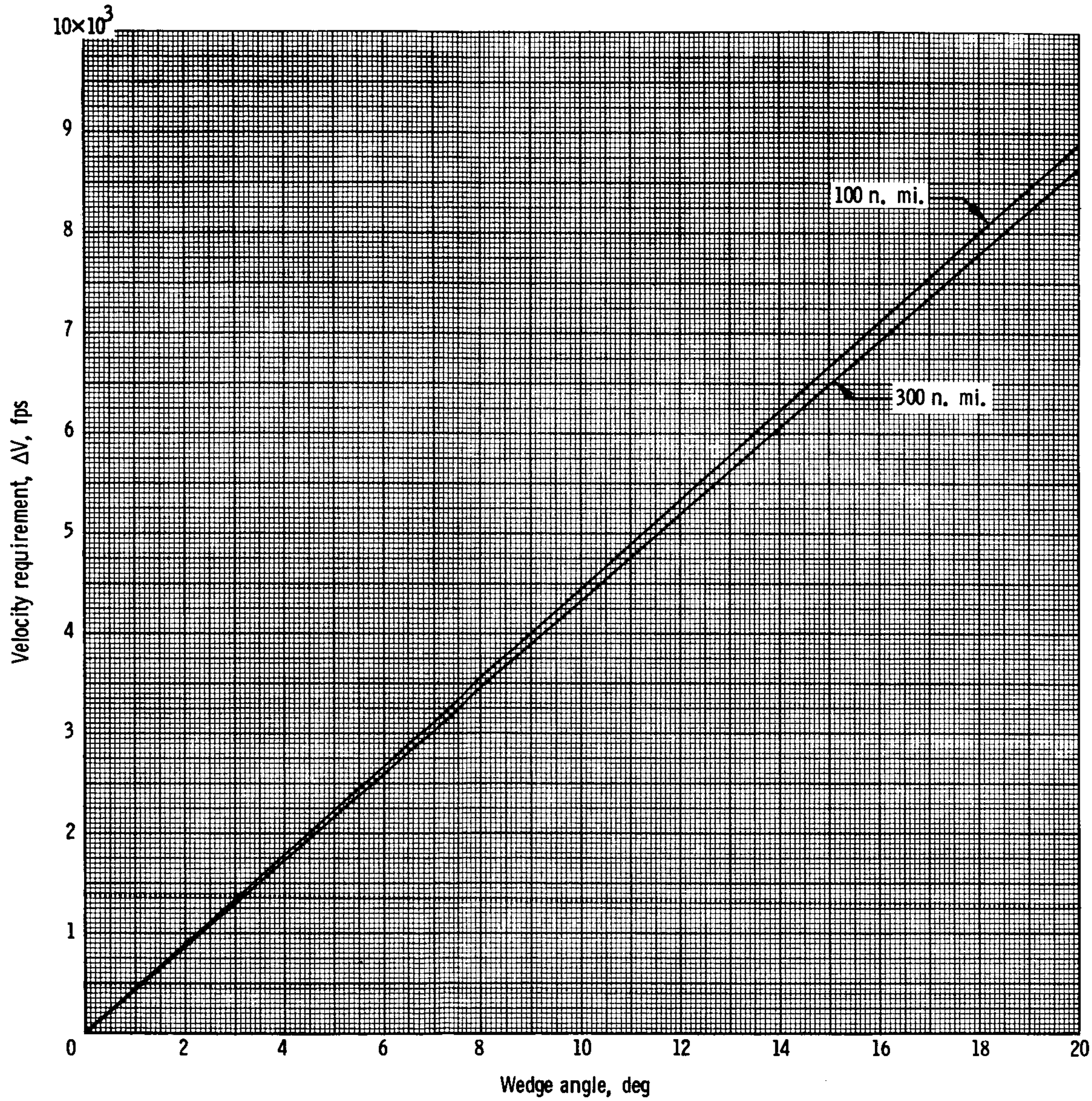


Figure 8. - Plane change velocity requirement versus wedge angle as a function of target altitude.